

# Perturbative QCD analysis of the Bjorken sum rule \*

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## Abstract

We study the polarized Bjorken sum rule at low momentum transfer squared  $Q^2 \leq 3$  GeV<sup>2</sup> in the twist-two approximation and to the next-to-next-to-leading order accuracy.

## 1 Introduction

The spin structure of a nucleon is one of the most interesting problems to be resolved within the framework of (nonperturbative) Quantum Chromodynamics (QCD). In particular, the singlet part  $\Sigma(x, Q^2)$  of the parton distribution functions

$$\Sigma(x, Q^2) = \sum_{i=1}^f f_a(x, Q^2),$$

where  $f$  is a number of active quarks, is intensively studied, because there is strong disagreement between the experimental data for its first Mellin moment and corresponding theoretical predictions. This disagreement is usually called a spin crisis (see, for example, reviews in [1]).

Here we consider only the non-singlet (NS) part, which the fundamental Bjorken sum rule (BSR) holds for [2]

$$\Gamma_1^{p-n}(Q^2) = \int_0^1 [g_1^p(x, Q^2) - g_1^n(x, Q^2)] dx.$$

It deals with the first moment ( $n = 1$ ) of NS part of the structure function (SF)  $g_1(x, Q^2)$ . For the case  $n = 1$ , the corresponding anomalous dimension of Wilson operators is zero and all the  $Q^2$ -dependence of  $\Gamma_1^{p-n}(Q^2)$  is encoded in the coefficient function.

Usually, BSR is represented in the form

$$\Gamma_1^{p-n}(Q^2) = \frac{g_A}{6} E_{NS}(Q^2) + \sum_{i=2}^{\infty} \frac{\mu_{2i}^{p-n}(Q^2)}{Q^{2i-2}},$$

where the first term in the r.h.s. is a twist-two part and the second one is a contribution of higher twists (HTs).

At high  $Q^2$  values the experiment data for  $\Gamma_1^{p-n}(Q^2)$  and the theoretical predictions [1] are well compatible with each other. Here we will focus on low  $Q^2$  values, at which there presently exist the very precise CLAS [3, 4] and SLAC [5] experimental data for BSR. On the other hand, there also is a great progress in theoretical calculations: recently, the terms  $\sim \alpha_s^4$  are evaluated in [6].

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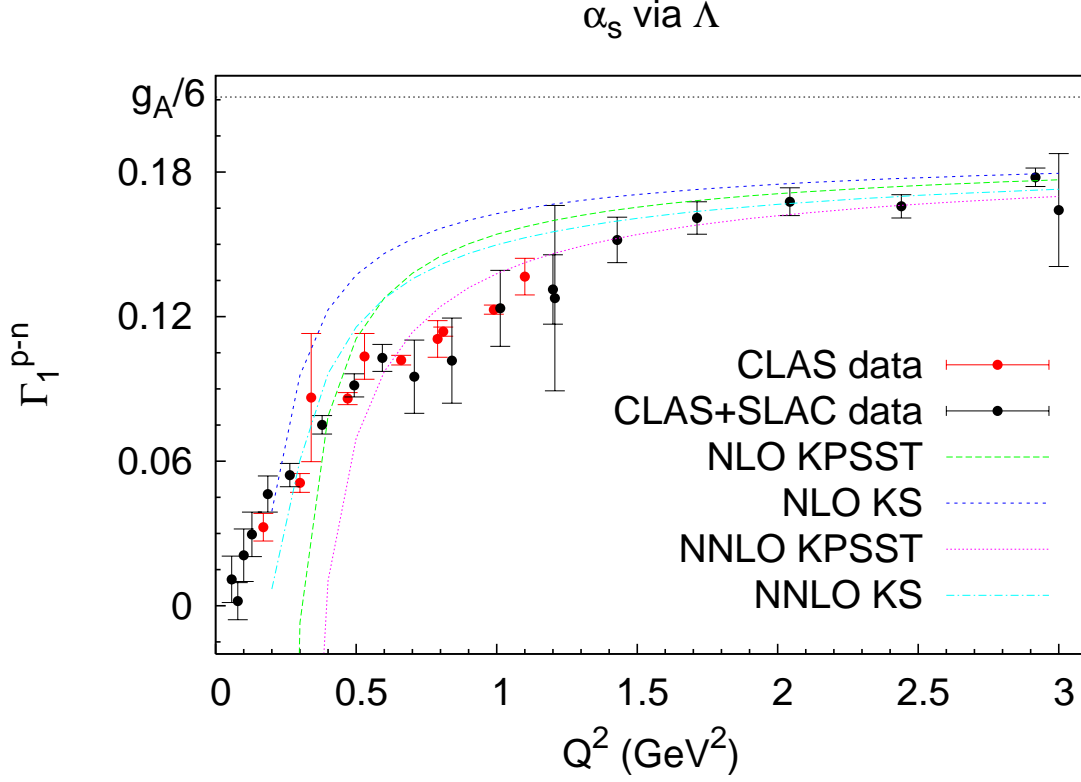


Figure 1: (color online). CLAS [3, 4] and SLAC [5] experimental data for BSR and  $Q^2 \leq 3$   $\text{GeV}^2$ . The curves represent theoretical predictions obtained in the analyzes carried out by two groups: Khandramai, Pasechnik, Shirkov, Solovtsova, and Teryaev (KPSST) [7] and Kotikov and Shaikhatdenov (KS).

## 2 Basic formulae

In our analysis we will mostly follow the analyses done by the Dubna-Gomel group [7, 8]. We try, however, to resum the twist-two part with the purpose of reducing a contribution coming from the HT terms.

Indeed, there is an interplay

- between HTs and higher orders of perturbative QCD corrections (see, for example, [9], where the SF  $xF_3$  was analyzed).
- between HTs and resummations in the twist-two part (see, for example, application of the Grunberg approach [10] in [11] to the study of SFs  $F_2$  and  $F_L$ )

The twist-two part of BSR has the following form (see, for example, [7])

$$E_{NS}(Q^2) = 1 - 4\Delta(Q^2), \quad (1)$$

where the term  $\Delta(Q^2)$  looks like

$$\Delta(Q^2) = a_s(Q^2) \left( 1 + \sum_{k=1}^{\infty} C_k a_s^k(Q^2) \right) \quad \left( a_s(Q^2) = \frac{\alpha_s(Q^2)}{4\pi} \right). \quad (2)$$

The first three coefficients  $C_1$ ,  $C_2$  and  $C_3$  are already known (see [6, 12] and references therein).

We will replace the above representation (2) by the following one

$$E_{NS}(Q^2) = \frac{1}{1 + 4\tilde{\Delta}(Q^2)}, \quad (3)$$

where

$$\tilde{\Delta}(Q^2) = a_s(Q^2) \left( 1 + \sum_{k=1}^{\infty} \tilde{C}_k a_s^k(Q^2) \right) \quad (4)$$

and  $\tilde{C}_k$  can be obtained from the known  $C_k$ :

$$\tilde{C}_1 = C_1 + 4, \quad \tilde{C}_2 = C_2 + 8C_1 + 16, \quad \tilde{C}_3 = C_3 + 8C_2 + 4C_1^2 + 48C_1 + 64. \quad (5)$$

The reason behind this transformation is as follows: the CLAS experimental data [3, 4] demonstrate that  $\Gamma_1^{p-n}(Q^2 \rightarrow 0) \rightarrow 0$ . Therefore, in the case when the HT corrections produce small contributions at  $Q^2 \rightarrow 0$ <sup>1</sup> we see that

$$E_{NS}(Q^2 \rightarrow 0) \rightarrow 0. \quad (6)$$

Since the strong coupling constant  $a_s(Q^2 \rightarrow \Lambda^2) \rightarrow \infty$ , it is seen that the form (3) behaves much like the CLAS experimental data. Indeed,

$$E_{NS}(Q^2 \rightarrow \Lambda^2) = \frac{1}{1 + 4\tilde{\Delta}(Q^2 \rightarrow \Lambda^2)} \rightarrow 0. \quad (7)$$

As  $\Lambda_{QCD}^2 \sim 0.01$  is rather small, one can conclude that the above representation (7) agrees with experiment at very low  $Q^2$  values.

Note, however, that we have a very small coefficients of  $\Delta(Q^2)$  and  $\tilde{\Delta}(Q^2)$ . Thus, for small but nonzero  $Q^2$  values the above representations (1) and (2) lead to similar results (see Fig. 1, where we restricted our consideration to the next-to-next-to-leading order (NNLO) accuracy). As is seen in Fig. 1, the theoretical predictions are not too close to the shape of the experimental data.

### 3 Grunberg approach

At  $Q^2 \sim 0$ , the value of the strong coupling constant is very large. Thus, in our approach it is better to avoid the usage of series like

$$\sum_{k=1}^{\infty} C_k a_s^k(Q^2), \quad (8)$$

as in Eqs. (2) and (4).

Instead, it is convenient to use the Grunberg method of effective charges [10], i.e. to consider the variables  $\Delta(Q^2)$  and  $\tilde{\Delta}(Q^2)$  as new effective “coupling constants”<sup>2</sup>, which have the following properties:

- shifted arguments

$$Q^2 \rightarrow Q^2/D_k, \quad Q^2 \rightarrow Q^2/\tilde{D}_k \quad (9)$$

for the variables  $\Delta(Q^2)$  and  $\tilde{\Delta}(Q^2)$ , respectively, with

$$D_k = e^{C_1/\beta_0}, \quad \tilde{D}_k = e^{\tilde{C}_1/\beta_0}, \quad (10)$$

which are in turn responsible for the vanishing of the coefficients  $C_1$  and  $\tilde{C}_1$  in a series similar to (8). Moreover, these shifted arguments (9) provide also a strong reduction in the magnitudes of the coefficients  $C_k$  and  $\tilde{C}_k$  ( $k \geq 2$ ).

- new  $\beta_i$  ( $i \geq 2$ ) coefficients of the corresponding  $\beta$ -functions, which are responsible for the vanishing of the coefficients  $C_k$  and  $\tilde{C}_k$  ( $k \geq 2$ ).

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<sup>1</sup>It is for sure questionable, but in the KPSST analysis [7]  $\mu_4 \sim 0$  at the next-to-next-to-next-to-leading order (N<sup>3</sup>LO) accuracy.

<sup>2</sup>A similar application can be found in the recent paper [13].

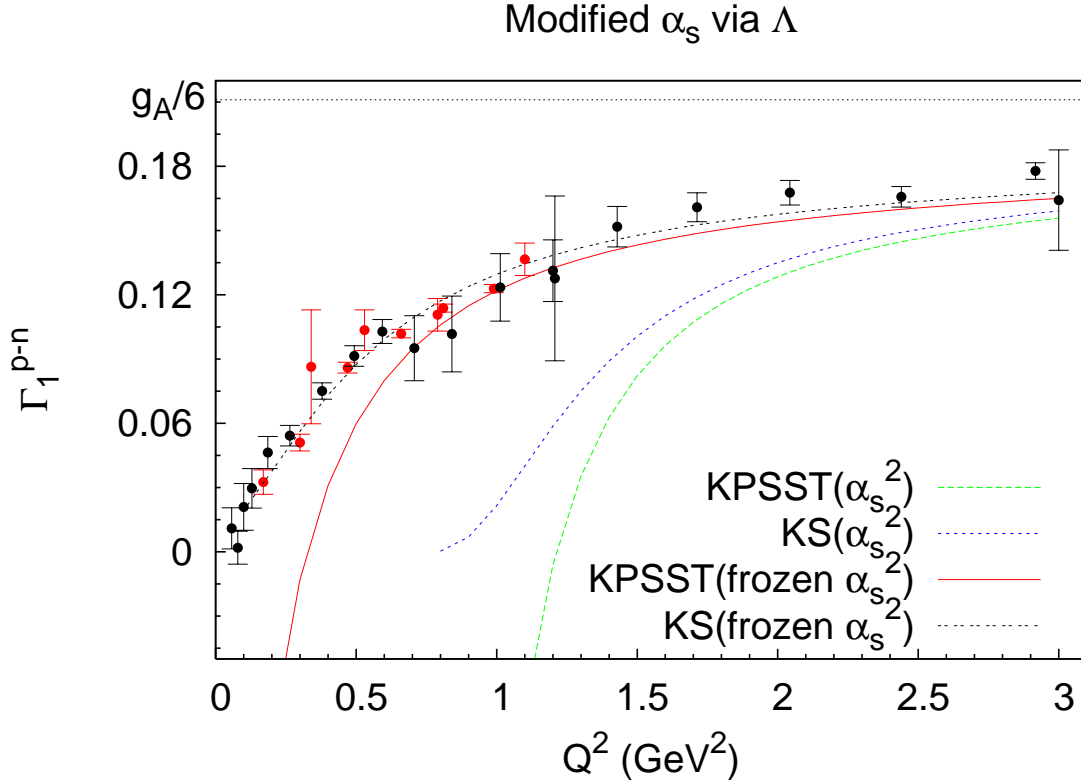


Figure 2: (color online). The experimental data are as in Fig.1. The curves show theoretical predictions based on Eqs. (1) and (3), which is called a KPSST-like analysis [7] and a KS one, respectively. For all cases the Grunberg approach [10] is used with a standard coupling constant and with a “frozen” one, when  $a = 1.5$ .

However, a straightforward application of the Grunberg approach to the variables  $\Delta(Q^2)$  and  $\tilde{\Delta}(Q^2)$  is not as convenient, because the coefficients  $C_1$  and  $\tilde{C}_1$  are positive and the  $Q^2$  values are very small. It is in contrast with its direct applications, where the coefficients  $C_1$  and  $\tilde{C}_1$  are negative [14] and/or the  $Q^2$  values are not so small [11, 15].

So, the new arguments  $Q^2/D_k$  and  $Q^2/\tilde{D}_k$  have now very small values and, as a result, we have to use the Grunberg approach associated with something else. One of the ways is to use a so-called “frozen” coupling constant.

## 4 “Frozen” coupling constant

We introduce freezing of the coupling constant by altering its argument  $Q^2 \rightarrow Q_a^2 = Q^2 + aM_\rho^2$ , where  $M_\rho$  is a  $\rho$ -meson mass and  $a$  is some free parameter (usually,  $a = 1$  was used. See, for example, [16]).

Thus, in the formulae of the previous sections the following replacement should be done (a list of references can be found in [17]):

$$a_s(Q^2) \rightarrow a_{fr}(Q^2) \equiv a_s(Q^2 + aM_\rho^2) \quad (11)$$

In the analyzes given below we restrict ourselves to the next-to-leading order (NLO) (i.e.  $\alpha_s^2$ ) approximation. The consideration of two even higher order corrections is in progress.

The cases with  $a = 1.5$  and  $a = 2$  are shown in Figs. 2 and 3, respectively. It is seen that the best agreement with experimental data is achieved in the case of representation

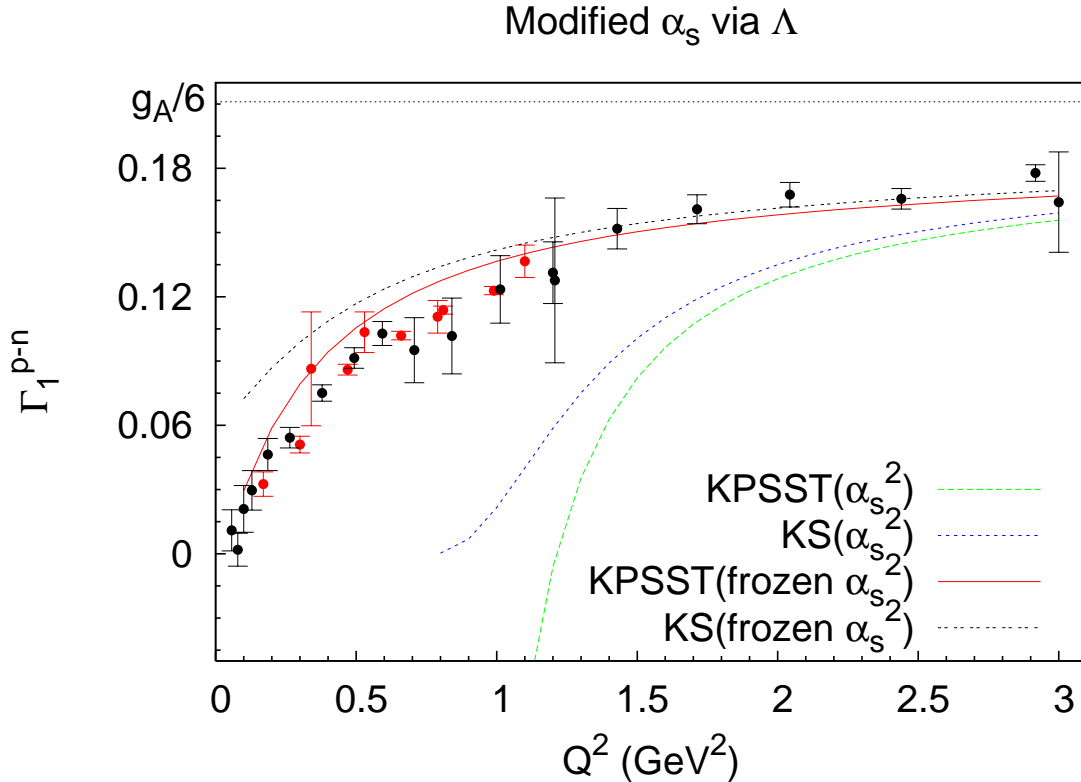


Figure 3: (color online). As in Fig 2 but  $a = 2$ .

(3) and  $a = 1.5$ . Also, the standard form (1) and theoretical predictions obtained with  $a = 2$  are well consistent with each other.

In contrast with the analyzes carried out with a standard coupling constant (see Fig. 1), we observe that the shape of theoretical predictions and the form of experimental data are close enough to each other at small  $Q^2$  values.

## 5 Conclusion

The analysis of the Bjorken sum rule performed within the framework of perturbative QCD is presented at low  $Q^2$ . It features the following important steps:

- The new form (3) for the twist-two part was used. It is compatible with the observation  $E_{NS}(Q^2 \rightarrow 0) \rightarrow 0$ , coming from the experimental data (if HTs are negligible).
- The application of the Grunberg method of effective charges [10] in a combination with a “frozen” coupling constant provides good agreement with experimental data, though with a slightly larger freezing parameter ( $1.5M_\rho^2$  instead of  $M_\rho^2$ ).<sup>3</sup>

Further elaborations to be undertaken include taking into account the  $\alpha_s^2$  and  $\alpha_s^3$  corrections to our analysis, as well as the study of HT corrections and their correlations with a freezing parameter  $a$  (in front of  $M_\rho^2$ ). We also plan to add to our analysis an analytic coupling constant [18], which has no the Landau pole and leads usually to the results, which are similar to those obtained in the case of the “frozen” coupling constant [17, 19].

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<sup>3</sup>It seems that the value of the parameter  $a$  depends on the order of perturbation theory. We plan to study this dependence in our forthcoming investigations.

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